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Quantitative Aptitude

Concept 1

2011

1. Number System
2. HCF and LCM

Prelims Paper II

NUMBER SYSTEM

- In Hindu Arabic System, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits to represent any number. This is the decimal system where we use the numbers 0 to 9.
- 0 is called insignificant digit.

Types of Numbers

- Natural numbers
- Whole numbers
- Integers
- Positive Integers
- Negative Integers
- Non-negative Integers
- Rational Numbers
- Irrational Numbers
- Real Numbers
- Even Numbers
- Odd Numbers
- Prime Numbers
- Composite Numbers

- **Natural numbers**
  - ✓ Counting numbers 1, 2, 3, 4, 5... are known as natural numbers.
  - ✓ The set of all natural numbers can be represented by N= {1, 2, 3, 4, 5…}

- **Whole numbers**
  - ✓ If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5 … are called whole numbers.
  - ✓ The set of whole number can be represented by W= {0, 1, 2, 3, 4, 5…}
  - ✓ Every natural number is a whole number but 0 is a whole number which is not a natural number.

- **Integers**
  - ✓ All counting numbers and their negatives including zero are known as integers.
  - ✓ The set of integers can be represented by Z or I = {…-4, -3, -2, -1, 0, 1, 2, 3, 4 …}

- **Positive Integers**
  - ✓ The set I^+ = {1, 2, 3, 4…} is the set of all positive integers.
  - ✓ Positive integers and natural numbers are synonyms.

- **Negative Integers**
  - ✓ The set I^- = {-1, -2, -3…} is the set of all negative integers.
  - ✓ 0 is neither positive nor negative.

- **Non-negative Integers**
  - ✓ The set {0, 1, 2, 3…} is the set all non-negative integers.

- **Rational Numbers**
  - ✓ The numbers of the form p/q, where p and q are integers and q ≠ 0, are known as rational numbers, e.g. 4/7, 3/2, -5/8, 0/1, -2/3, etc.
  - ✓ The set of all rational numbers is denoted by Q. i.e. Q ={x:x =p/q; p, q belong to I, q≠0}.
  - ✓ Since every natural number 'a' can be written as a/1, every natural number is a rational number.
  - ✓ Since 0 can be written as 0/1 and every non-zero integer 'a' can be written as a/1, every integer is a rational number.
  - ✓ Every rational number has a peculiar characteristic that when expressed in decimal form is expressible rather in terminating decimals or in non-terminating repeating decimals.
  - ✓ For example, 1/5 =0.2, 1/3 = 0.333…22/7 = 3.1428704287, 8/44 = 0.181818…., etc.
  - ✓ The recurring decimals have been given a short notation as

    $0.333.... = 0.\overline{3}$

    $4.1555... = 4.0\overline{5}$

    $0.323232... = 0.\overline{3}2.$

- **Irrational Numbers**
  - ✓ Those numbers which when expressed in decimal from are neither terminating nor repeating decimals are known as irrational number, e.g. √2, √3, √5, π, etc.
  - ✓ Note that the exact value of π is not 22/7. 22/7 is rational while π irrational number. 22/7 is approximate value of π. Similarly, 3.14 is not an exact value of it.

- **Real Numbers**
  - ✓ The rational and irrational numbers combined together are called real numbers, e.g.13/21, 2/5, -3/7; √3, 4 + √2, etc. are real numbers.
  - ✓ The set of all real numbers is denoted by R.
  - ✓ Note that the sum, difference or product of a rational and irrational number is irrational, e.g. 3+ √2, 4-√3, 2/3-√5, 4√3, -7√5 are all irrational.

- **Even Numbers**
  - ✓ All those numbers which are exactly divisible by 2 are called even numbers, e.g.2, 6, 8, 10, etc., are even numbers.

- **Odd Numbers**
  - ✓ All those numbers which are not exactly divisible by 2 are called odd numbers, e.g. 1, 3, 5, 7 etc., are odd numbers.

- **Prime Numbers**
  - ✓ A natural number other than 1 is a prime number if it is divisible by 1 and itself only.
  - ✓ For example, each of the numbers 2, 3, 5, 7 etc., are prime numbers.

- **Composite Numbers**
  - ✓ Natural numbers greater than 1which are not prime, are known as composite numbers.
  - ✓ For example, each of the numbers 4, 6, 8, 9, 12, etc., are composite numbers.

BITS

The number 1 is neither a prime number nor composite number.

2 is the only even number which is prime

Prime numbers up to 100 are: 2, 3, 5, 7, 11, 13, 17, 19,23, 29, 31, 37, 41,43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, i.e. 25 prime numbers between 1 and 100.

Two numbers which have only 1 as the common factor are called co-primes or relatively prime to each other, e.g. 3 and 5 are co-primes.

H.C.F. and L.C.M.

Common factor

A common factor of two or more numbers is a number which divides each of them exactly.

For example, 4 is a common factor of 8 and 12.

Highest common factor (H.C.F.)

Highest common factor of two or more numbers is the greatest number that divides each one of them exactly.

For example, 6 is the highest common factor of 12, 18 and 24.

Highest Common Factor is also called Greatest Common Divisor or Greatest Common Measure. Symbolically, these can be written as H.C.F. or G.C.D. or G.C.M., respectively.

Methods of Finding H.C.F.

I. Method of Prime Factors

Step 1

Express each one of the given numbers as the product of prime factors.

[A number is said to be a prime number if it is exactly divisible by 1 and itself but not by any other number, e.g. 2, 3, 5, 7, etc. are prime numbers]

Step 2

Choose Common Factors.

Step 3

Find the product of lowest powers of the common factors. This is the required H.C.F. of given numbers.

Example 1

Find the H.C.F. of 70 and 90.

Solution

$70 = 2 \times 5 \times 7$

$90 = 2 \times 5 \times 9$

Common factors are 2 and 5.

$\therefore$ H.C.F. = $2 \times 5 = 10$.

Example 2

Find the H.C.F. of 3332, 3724 and 4508

Solution

$3332 = 2 \times 2 \times 7 \times 7 \times 17$

$3724 = 2 \times 2 \times 7 \times 7 \times 19$

$4508 = 2 \times 2 \times 7 \times 7 \times 23$

$\therefore$ H.C.F. = $2 \times 2 \times 7 \times 7 = 196$.

Example 3

Find the H.C.F. of 360 and 132.

Solution

$360 = 2^3 \times 3^2 \times 5$

$132 = 2^2 \times 3^1 \times 11$

$\therefore$ H.C.F. = $2^2 \times 3^1 = 12$.

Example 4

If $x = 2^3 \times 3^5 \times 5^9$ and $y = 2^5 \times 3^7 \times 5^{11}$, find H.C.F. of x and y.

Solution

The factors common to both x and y are 2^3, 3^5 and 5^9.

$\therefore$ H.C.F. = $2^3 \times 3^5 \times 5^9$.

II. Method of Division

A. For two numbers:

Step 1

Greater number is divided by the smaller one.

Step 2

Divisor of (1) is divided by its remainder.

Step 3

Divisor of (2) is divided by its remainder. This is continued until no remainder is left.

H.C.F. is the divisor of last step.

Example 5

Find the H.C.F. of 3556 and 3444.

Solution

3444)3556 (1

3444

112) 3444 (30

3360

84) 112 (1

84

28) 84 (3

84

×

$\therefore$ HCF=Divisor of last step=28

B. For more than two numbers:

Step 1

Any two numbers are chosen and their H.C.F. is obtained.

Step 2

H.C.F. of H.C.F. (i.e. HCF obtained in step 1) and any other number is obtained.

Step 3

H.C.F. of H.C.F. (i.e. HCF obtained in last step) and any other number (not chosen earlier) is obtained.

This process is continued until all numbers have been chosen. H.C.F. of last step is the required H.C.F.

Example 6

Find the greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm and 12 m 95 cm

Solution

Required length = HCF of 7 m, 3 m 85 cm and 12 m 95 cm

=HCF of 700 cm, 385 cm and 1295 cm

= 35 cm.

Common Multiple

A common multiple of two or more numbers is a number which is exactly divisible by each one of them.

For Example, 32 is a common multiple of 8 and 16.

$8 \times 4 = 32$

$16 \times 2 = 32$

Least Common Multiple

The least common multiple of two or more given numbers is the least or lowest number which is exactly divisible by each of them.

For example, consider the two numbers 12 and 18.

Multiples of 12 are 12, 24, 36, 48, 72 …

Multiple of 18 are 18, 36, 54, 72 …

Common multiples are 36, 72 …

$\therefore$ Least common multiple, i.e. L.C.M. of 12 and 18 is 36.

Methods of Finding L.C.M.

I. Method of Prime Factors

Step 1

Resolve each given number into prime factors.

Step 2

Take out all factors with highest powers that occur in given numbers.

Step 3

Find the product of these factors. This product will be the L.C.M.

Example 7

Find the L.C.M. of 32, 48, 60 and 320.

Solution

32 = 2x2x2x2x2x1

48 = 2x2x2x2x3

60 = 2x2x3x5

320 = 2x2x2x2x2x2x5

∴ L.C.M. = 2x2x2x2x2x2x3x 5 = 960.

II. Method of Division

Step 1

The given numbers are written in a line separated by common.

Step 2

Divide by any one of the prime numbers 2, 3, 5, 7, 11 … which will divide at least any two of the given nu8mbers exactly. The quotients and the undivided numbers are written in a line below the first.

Step 3

Step 2 is repeated until a line of numbers (prime to each other) appears.

Find the product of all divisors and numbers in the last line which is the required L.C.M.

Example 8

Find the L.C.M. of 12, 15, 20 and 54.

Solution

2	**12**	**15**	**20**	**54**
2	6	15	10	27
3	3	15	5	27
3	1	5	1	9
3	1	5	1	3
5	1	5	1	1
	1	1	1	1

∴ L.C.M. = $2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$

Note: Before finding the L.C.M. or H.C.F., we must ensure that all quantities are expressed in the same unit.

Tips

1. H.C.F. and L.C.M. of Decimals

Make the same number of decimal places in all the given numbers by suffixing zero(s) if necessary.

Find the H.C.F. /L.C.M. of these numbers without decimal.

Put the decimal point (in the H.C.F. /L.C.M. of step 2) leaving as many digits on its right as there are in each of the numbers.

2. L.C.M. and H.C.F. of Fractions

$$L.C.M. = \frac{L.C.M. \text{ of the numbers in numerators}}{H.C.F. \text{ of the numbers in denominators}}$$

$$H.C.F. = \frac{H.C.F. \text{ of the numbers in numerators}}{L.C.M. \text{ of the numbers in denominators}}$$

3. Product of two numbers

= L.C.M. of the numbers $\times$ H.C.F. of the numbers

4. To find the greatest number that will exactly divide x, y and z.

Required number = H.C.F. of x, y and z

5. To find the greatest number that will divide x, y and z leaving remainders a, b and c, respectively.

Required number = H.C.F. of $(x - a)$, $(y - b)$ and $(z - c)$

6. To find the least number which is exactly divisible by x, y and z.

Required number = L.C.M. of x, y and z

7. To find the least number which when divided by x, y and z leaves the remainders a, b and c, respectively.

It is always observed that (x – a) = (y – b) = (z – c) = k (say)

$\therefore$ Required number = (L.C.M. of x, y and z) – k.

8. To find the least number which when divided by x, y and z leaves the same remainder r in each case.

Required number = (L.C.M. of x, y and z) + r.

9. To find the greatest number that will divide x, y and z leaving the same remainder in each case.

- ✓ When the value of remainder r is given:
  Required number = H.C.F. of $(x - r)$, $(y - r)$ and $(z - r)$.
- ✓ When the value of remainder is not given:
  Required number = H.C.F. of l(x – y) l, l(y – z) l and l (z – x) l

10. To find the n-digit greatest number which, when divided by x, y and z.

- ✓ Leaves no remainder (i.e. exactly divisible)

  Step 1: L.C.M. of x, y and $z = L$

  L) n digit greatest number (

  Step 2: remainder = R

  Step 3: Required number

  = n digit greatest number – R

- ✓ leaves remainder K in each case Required number
  = (n digit greatest number – R) + K.

11. To find the n-digit smallest number which when divided by x, y and z

- ✓ leaves no remainder (i.e. exactly divisible)

  Step 1: L.C.M. of x, y and z = L

  L) n-digit smallest number (

  Step 2: remainder = R

  Step 3: Required number

  = n-digit smallest number + $(L - R)$.

- ✓ Leaves remainder K in each case.
  Required number = n-digit smallest number + $(L - R) + k$.